QUANTUM MECHANICS

→ PHOTOELECTRIC EFFECT: (corpscular nature → Particle nature)

1> Quantum of light: Photon

energy of each photon: $E = h\nu$

$h \rightarrow$ Plank's constant

$\nu \rightarrow$ frequency of Photon (Hz)

Properties of Photon:

→ mass = zero
→ charge = zero
→ Q.No.'s = zero

→ not deflected in Electric & magnetic field
→ $E = pc$

→ called as a virtual Particle

2> Defination: Phenomenon of emission of electrons by the metals when they are exposed to light of suitable frequency is called photoelectric effect. (K-shell $e^{\ominus}$)

→ Hertz experiment
→ Provides a direct confirmation of energy quantization of light

3> Characteristics:

→ Ejection of electrons: $\nu_{incident} \geq \nu_{threshold}$
→ maximum K.E of ejected electrons: $K_{max} = h\nu - \phi$
→ workfunction is minimum energy required to eject the electrons. $\phi = h\nu_0$ (work fxn ≃ Ionization energy)

For metals $\phi = 2\text{-}6$ eV

$\boxed{(K.E)_{max} \propto \text{frequency}}$ $\quad (K.E)_{max} = h(\nu - \nu_0)$

→ Photocurrent: $I_p = \frac{dq}{dt} = \frac{d}{dt}(ne) = e\frac{dn}{dt}$

$\boxed{\text{Photocurrent} \propto \text{Intensity of light}}$

→ Instantaneous Process: no time difference between the incident Radiation & ejection of electrons

→ Stopping Potential: $E = e|V|_s = h\nu - \phi$

$$|V|_s = \frac{h\nu}{e} - \frac{\phi}{e} \Rightarrow \frac{hc}{e\lambda} - \frac{\phi}{e}$$

→ Superposition of two waves:

$$y_1 = A\cos(K_1x - \omega_1 t) \; ; \; y_2 = A\cos(K_2x - \omega_2 t)$$

$$y = y_1 + y_2 = A\left[\cos(K_1x - \omega_1 t) + \cos(K_2x - \omega_2 t)\right]$$

$$y = 2A\cos\left[\left(\frac{K_1+K_2}{2}\right)x - \left(\frac{\omega_1+\omega_2}{2}\right)t\right]\cos\left[\left(\frac{K_1-K_2}{2}\right)x - \left(\frac{\omega_1-\omega_2}{2}\right)t\right]$$

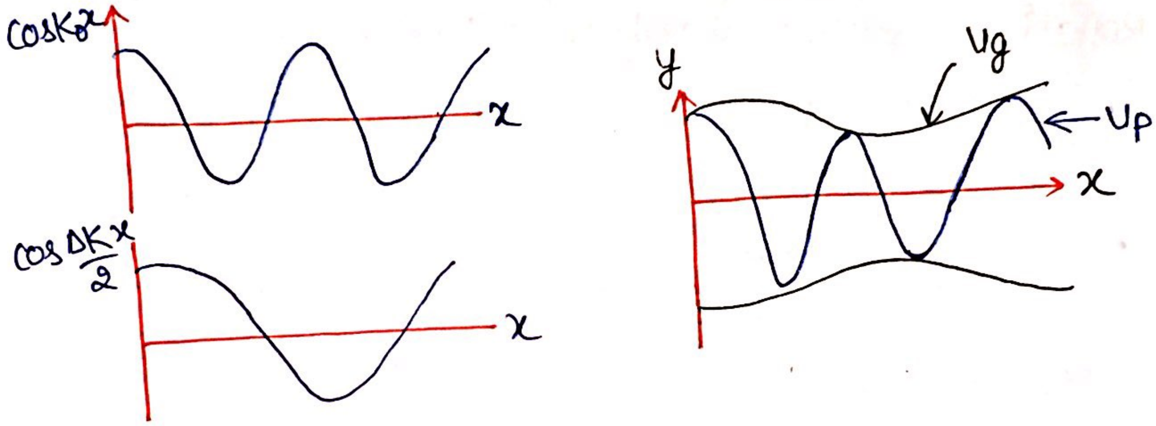
let $\frac{K_1+K_2}{2} = K_0$, $\frac{\omega_1+\omega_2}{2} = \omega_0$; $K_1 - K_2 = \Delta x$

$\omega_1 - \omega_2 = \Delta \omega$

$$y = 2A\cos(K_0x - \omega_0 t)\cos\left(\frac{\Delta K x}{2} - \frac{\Delta\omega}{2}t\right)$$

at $t=0$:

$$y = 2A\cos K_0 x \cos\frac{\Delta K}{2}x$$



Phase of wave always constant:

1) $K_0 x - \omega_0 t = \text{constant}$ 2) $\frac{\Delta K}{2}x - \frac{\Delta\omega}{2}t = \text{constant}$

$$\boxed{\frac{dx}{dt} = \frac{\omega_0}{K_0}} \rightarrow \text{Phase velocity} \qquad \boxed{\frac{dx}{dt} = \frac{d\omega}{dK}} \rightarrow \text{group velocity}$$

1) Phase velocity:

$$v_p = \frac{\omega}{K} = \frac{\hbar\omega}{\hbar K}$$

$$\boxed{v_p = \frac{\omega}{K} = \frac{E}{P}}$$

2) Group velocity:

$$v_g = \frac{d\omega}{dK} = \frac{d(\hbar\omega)}{d(\hbar K)}$$

$$\boxed{v_g = \frac{d\omega}{dK} = \frac{dE}{dP}}$$

Examples:

① A 50 KW broadcasting antenna emits radio waves at a frequency of 5MHz. Find how many photons are emitted per second?

a) 1.8×10^{31} b) 1.5×10^{31} c) 1.5×10^{30} d) 1.8×10^{30}

Soln:- $E = nh\nu$

$$n = \frac{E}{h\nu} = \frac{50\times10^{3}}{6.626\times10^{-34}} \times \frac{1}{5\times10^{6}}$$

$$n = 1.5\times10^{31}$$

Hence, option (b) is CORRECT

② When light of a given wavelength is incident on a metallic surface, the stopping potential for the photo-electrons is 3.2 V. If a second light source whose wavelength is double that of the first is used, the stopping potential drops to 0.8 V. From these data, calculate the wavelength of first radiation:

a) 2.6×10^{-7}m b) 1.3×10^{-7}m c) 2.6×10^{-6}m d) 1.3×10^{-6}m

Soln:- given: $\lambda_2 = 2\lambda_1$

$$V_{s_1} = \frac{hc}{e\lambda_1} - \frac{W}{e} \quad \& \quad V_{s_2} = \frac{hc}{e\lambda_2} - \frac{W}{e}$$

$$V_{s_1} - V_{s_2} = \frac{hc}{e}\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right) = \frac{hc}{e\lambda_1}\left(1 - \frac{1}{2}\right)$$

$$\lambda_1 = \frac{hc}{2e(V_{s_1} - V_{s_2})} = \frac{6.6\times10^{-34}\times3\times10^{8}}{2\times1.6\times10^{-19}\times(3.2-0.8)}$$

$$\lambda_1 = 2.6\times10^{-7}\text{m}$$

Hence, option (a) is CORRECT

also commute.

i.e $[\hat{A}, f(\hat{A})] = 0$ and $[f_1(\hat{A}), f_2(\hat{A})] = 0$

6) $[\hat{A}, \hat{B}^n] = n\hat{B}^{n-1}[\hat{A}, \hat{B}]$

If $[\underbrace{[\hat{A}, \hat{B}]}_{\text{must be scaler}}, \hat{B}] = 0$

7) Jacobi Identity :

$$[[\hat{A}, \hat{B}], \hat{C}] + [[\hat{B}, \hat{C}], \hat{A}] + [[\hat{C}, \hat{A}], \hat{B}] = 0$$

Generalised expression for Heisenberg uncertainity Principle:

$$(\Delta A)^2 (\Delta B)^2 \geq -\frac{1}{4}(\langle[\hat{A}, \hat{B}]\rangle)^2$$

8) Parity operator: an operator that reflects the space about origine is called parity operator.

$$\boxed{\hat{P}\psi(r) = \psi(-r)}$$

→ Hermitian operator : $\hat{P}^{+} = \hat{P}$

→ Involumentory : $\hat{P}^2 = \hat{I}$

→ eigen values : $\lambda = \pm 1$

→ Commutes with Hamiltonian: $\underbrace{[\hat{P}, \hat{H}] = 0}_{\text{as for symmetrical Potential}}$

→ unitary operator:

$\hat{P}^{+}\hat{P} = \hat{I}$; $\hat{P}^{+} = \hat{P}^{-1}$

$\psi(0) = 0 \Rightarrow B = 0$

$\psi(L) = 0 \Rightarrow k = \left(\frac{n\pi}{L}\right)$ also $k = \frac{\sqrt{2mE}}{\hbar}$

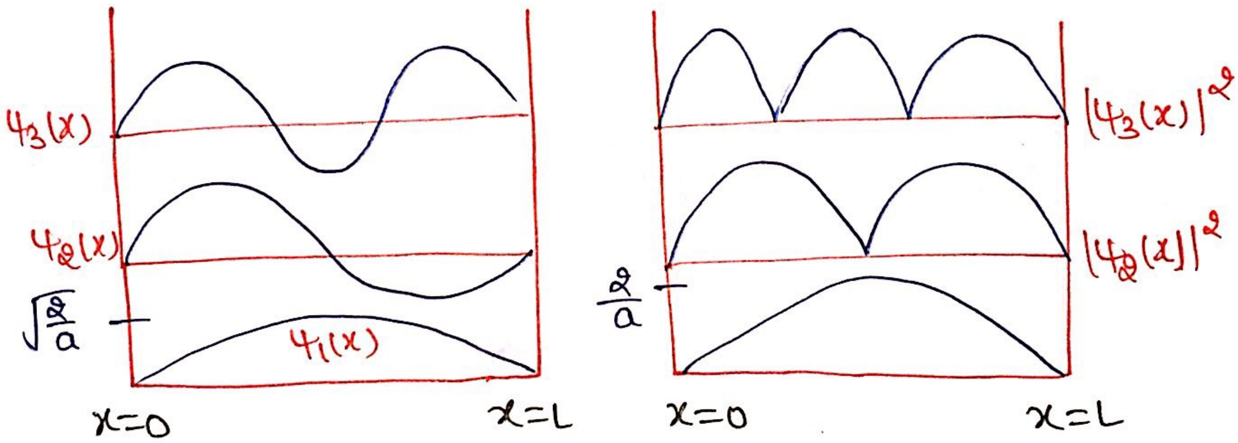
$$\boxed{\psi_n = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right)} \rightarrow \text{normalized wavefunction}$$

$$\boxed{E_n = \frac{n^2\pi^2\hbar^2}{2mL^2} = n^2 E_1} \rightarrow \text{energy in 1-D}$$

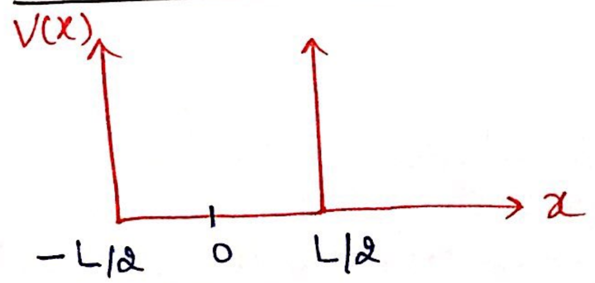
For time Dependent Schrodinger eqn:

$$\psi(x,t) = \sum_{n=1}^{\infty} \psi_n(x)\, e^{-iE_n t/\hbar}$$

$$= \sqrt{\frac{2}{L}} \sum_{n=1}^{\infty} \sin\left(\frac{n\pi x}{L}\right) e^{-i n^2 E_1 t/\hbar}$$



→ asymmetric square well Potential 1-Dim:



$\text{width} = \frac{L}{2} - \left(-\frac{L}{2}\right) = \frac{2L}{2} \Rightarrow L$

$V(-x) = V(x)$

symm. Potential

$\psi = Ae^{ikx} + Be^{-ikx}$

$$r^2 = \frac{2m V_0 (\text{+ve width})^2}{\hbar^2} = \frac{2mV_0 (a/2)^2}{\hbar^2}$$

$$r^2 = \frac{mV_0 a^2}{2\hbar^2}$$

$$(n-1)\pi/2 < r < n\pi/2$$

$$(3-1)\pi/2 < r < 3\pi/2$$

$$\pi^2 < r^2 < \frac{9}{4}\pi^2$$

$$\pi^2 < \frac{mV_0 a^2}{2\hbar^2} < \frac{9}{4}\pi^2$$

$$\boxed{\frac{2\pi^2\hbar^2}{ma^2} < V_0 < \frac{9\pi^2\hbar^2}{2ma^2}}$$

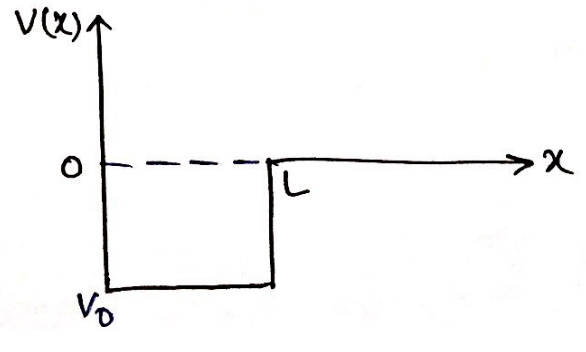
Que 2 A particle in 1-Dim. is in Potential

$$V_x = \begin{cases} \infty & \text{if } x < 0 \\ V_0 & 0 \le x \le l \\ 0 & x > l \end{cases}$$

If there is atleast one bound state, the minimum depth of the potential is:

a) $\frac{\hbar^2\pi^2}{8ml^2}$ b) $\frac{\hbar^2\pi^2}{2ml^2}$ c) $\frac{2\hbar^2\pi^2}{ml^2}$ d) $\frac{\hbar^2\pi^2}{ml^2}$

Solnᵒ:-



3-Dimensional Harmonic Oscillator:

Anisotropic (Not symm.)	Isotropic (symmetric)
$\omega_x \neq \omega_y \neq \omega_z$	$\omega_x = \omega_y = \omega_z$
$E_n = (n_x+\frac{1}{2})\hbar\omega_x + (n_y+\frac{1}{2})\hbar\omega_y + (n_z+\frac{1}{2})\hbar\omega_z$	$E_n = (n_x+n_y+n_z+\frac{3}{2})\hbar\omega = (n+\frac{3}{2})\hbar\omega$, $n = n_x+n_y+n_z$
non-degenerate	$\boxed{\text{Deg.} = \frac{(n+1)(n+2)}{2}}$ ***

Que1 A 1-Dim. harmonic oscillator is in the state

$$\psi(x) = \frac{1}{\sqrt{14}}\left[3\psi_0(x) - 2\psi_1(x) + \psi_2(x)\right]$$

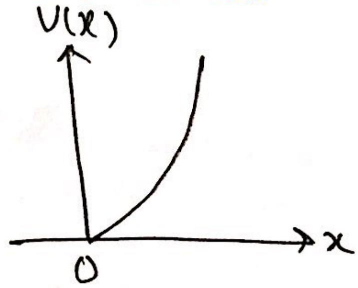
The probability of finding the oscillator in the ground state is:

a) 0 b) $3/\sqrt{14}$ c) $9/14$ d) 1

Solnᵒ:- $P[|\psi_0(x)|^2] = \left|\left(\frac{3}{\sqrt{14}}\right)\right|^2 = \frac{9}{14}$ $\boxed{c}$

Que2 A particle of mass m confined in the potential

$$V(x) = \begin{cases} \frac{1}{2}m\omega^2 x^2 & x > 0 \\ \infty & x \leq 0 \end{cases}$$



Let the wavefxn is:

$$\psi(x) = -\frac{1}{\sqrt{5}}\psi_0 + \frac{2}{\sqrt{5}}\psi_1$$

a) $\langle x\rangle = \sqrt{\frac{\hbar}{m\omega}}$, $c_0 = \frac{1}{\sqrt{2}}$ b) $\langle x\rangle = \sqrt{\frac{\hbar}{2m\omega}}$, $c_0 = \frac{1}{2}$

c) $\langle x\rangle = \sqrt{\frac{\hbar}{2m\omega}}$, $c_0 = \frac{1}{\sqrt{2}}$ d) $\langle x\rangle = \sqrt{\frac{\hbar}{m\omega}}$, $c_0 = \frac{1}{2}$

Solⁿ:- $|\psi\rangle = c_0|0\rangle + c_1|1\rangle$

$|c_0|^2 + |c_1|^2 = 1$ → For linear combination

$$\langle x\rangle = \langle\phi|x|\phi\rangle$$

$$= \left[\langle 0|c_0 + \langle 1|c_1\right]\left[\sqrt{\frac{\hbar}{2m\omega}}(a+a^\dagger)\right]\left[c_0|0\rangle + c_1|1\rangle\right]$$

$$\langle x\rangle = \sqrt{\frac{\hbar}{2m\omega}}\Big[c_0\langle 0|\cancel{a+a^\dagger}|0\rangle^{0} + c_1\langle 1|\cancel{a+a^\dagger}|1\rangle^{0}$$

$$+ c_0c_1\langle 0|a+a^\dagger|1\rangle + c_1c_0\langle 1|a+a^\dagger|0\rangle\Big]$$

$$\langle x\rangle = 2c_0c_1\sqrt{\frac{\hbar}{2m\omega}}$$

↑

maximum when $c_0 = c_1 = 1/\sqrt{2}$

$$c_0c_1 = 1/2$$

$$\langle x\rangle = \sqrt{\frac{\hbar}{2m\omega}}$$ $\boxed{C}$

Que 4 Let $a = \frac{1}{\sqrt{2}}(x+ip)$ and $a^\dagger = \frac{1}{\sqrt{2}}(x-ip)$ be the lowering and raising operators of a simple

→ **Spin Angular momentum:**

existence of spin was confirmed experimentally By stern and Gerlach.

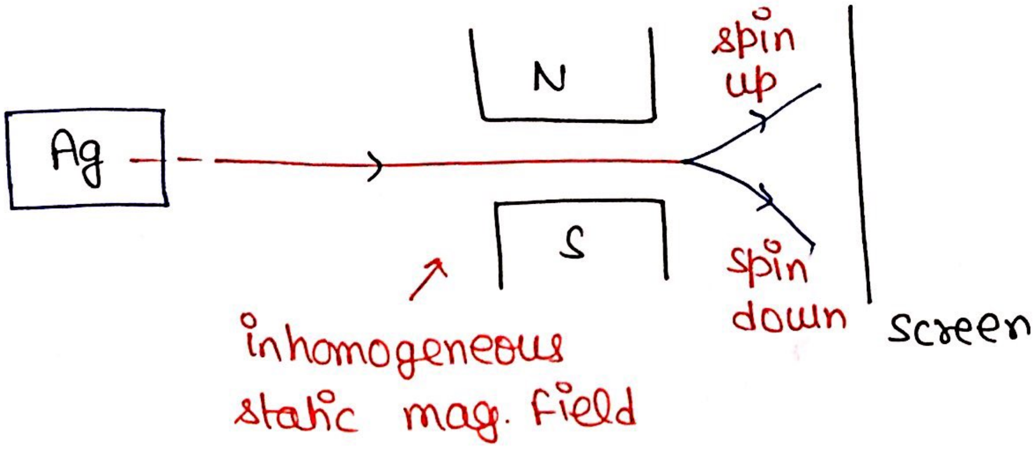


Fig: stern-Gerlach experiment

Silver (Ag) : $47 e^{\ominus}s$ → 46 are spherically symmetric ; → occupies 5s orbital (47^{th})

$e^{\ominus}$ beam behaves neither classical nor schrodinger wave theory. Instead it splits into two different components.

*** → Spin: intrinsic DOF, purely Q.M concept.

General theory of spin:

identical to general theory of angular momentum.

1) $[\hat{S}_x, \hat{S}_y] = i\hbar S_z$; $[\hat{S}_x, \hat{S}_z] = -i\hbar S_y$

2) $[S^2, S_z] = 0$ commute

→ $S^2|s, m_s\rangle = s(s+1)\hbar^2 |s, m_s\rangle$

→ $S_z|s, m_s\rangle = m_s\hbar |s, m_s\rangle$

ADDITION OF ANGULAR MOMENTA

→ addition of spin: $S = S_1 + S_2$

→ addition of L & S: $J = L + S$

→ addition of angular momenta: $J = J_1 + J_2$

Note: all commutation relations are same as in orbital momentum.

C.G coefficients

$$\underbrace{|s, m_s\rangle}_{\uparrow} = \sum_{m_{s_1}, m_{s_2}} \underbrace{\langle s_1 s_2 | m_{s_1}, m_{s_2}\rangle}_{\uparrow} \underbrace{\langle s_1 m_{s_1} | s_2 m_{s_2}\rangle}_{\uparrow}$$

Coupled states — square matrix — uncoupled states

Example: If $s_1 = s_2 = 1/2$ & $m_{s_1} = m_{s_2} = \pm 1/2$

$$\begin{bmatrix} |1,1\rangle \\ |1,0\rangle \\ |1,-1\rangle \\ |0,0\rangle \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1/\sqrt{2} & -1/\sqrt{2} & 0 \end{bmatrix}_{4\times4} \begin{bmatrix} |\frac{1}{2},\frac{1}{2}\rangle & |\frac{1}{2},\frac{1}{2}\rangle \\ |\frac{1}{2},\frac{1}{2}\rangle & |\frac{1}{2},-\frac{1}{2}\rangle \\ |\frac{1}{2},-\frac{1}{2}\rangle & |\frac{1}{2},-\frac{1}{2}\rangle \\ |\frac{1}{2},-\frac{1}{2}\rangle & |\frac{1}{2},\frac{1}{2}\rangle \end{bmatrix}$$

Que! If $s_1 = s_2 = 1$, the find coupled and uncoupled states?

Solⁿ:- $m_{s_1} = m_{s_2} = 1, 0, -1$

uncoupled states:

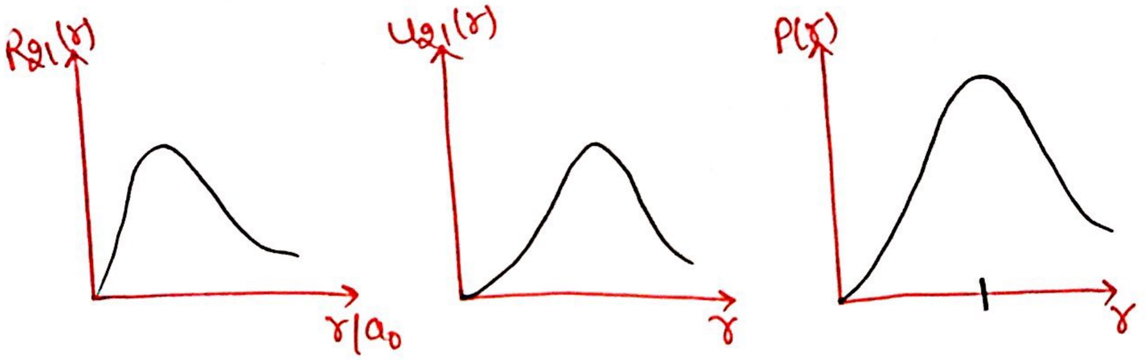
$\|s_1\ m_{s_1}\rangle = \|1,1\rangle$	$\|s_2\ m_{s_2}\rangle = \|1,1\rangle$
$\|1,0\rangle$	$\|1,0\rangle$
$\|1,-1\rangle$	$\|1,-1\rangle$

2) ψ_{210}: $n=2, l=1, m=0$

$j_{max} = n-l-1 = 0$

$R_{10} = \frac{1}{\sqrt{24a_0^3}} \frac{r}{a} \exp\left(-\frac{r}{2a_0}\right)$

central node: Present



Commutation Relation of H-atom in spherical coordinates

$\langle H \rangle = \langle T \rangle + \langle V \rangle$

$= \langle T \rangle - 2\langle T \rangle \Rightarrow -\langle T \rangle$

if $H|\psi\rangle = E|\psi\rangle$

$E_n = -\frac{1}{2m}\langle p^2 \rangle \Rightarrow \boxed{\langle p^2 \rangle = -2mE_n}$

$p^2 = -\hbar^2 \nabla^2 \quad (\because p = -i\hbar\nabla)$

$\boxed{p^2 = p_r^2 + \frac{L^2}{r^2}} \qquad \boxed{p_r = -i\hbar\left(\frac{\partial}{\partial r} + \frac{1}{r}\right)}$

Self adjoint operator (Hermitian)

$[p_r, H] \begin{cases} = \frac{i\hbar}{m}\left(\frac{L^2}{r^3}\right) - i\hbar \frac{\partial V}{\partial r} \\ = 0 \text{ if } p_r \text{ is time independent} \end{cases}$

$\left[p_r, \frac{L^2}{r^2}\right] = 2i\hbar\,(L^2/r^3)$

→ **Symmetric characteristic doesn't change w.r.t time:**

1> If system is symmetric at a particular time then it will be symmetric at any later time.

$$H\psi = i\hbar \frac{\partial \psi}{\partial t}$$

i.e Hamiltonian is always symmetric

2> If system is antisymmetric at a particular time then it will be antisymmetric at any later time.

→ **Spin States:**

For N-particles, total no. of spin states $= (2s+1)^N$

e.g. For $N=2$ & $s=1/2$

no. of spin states $= (2\times\frac{1}{2}+1)^2 = 4$

$|s, m_s\rangle \;=\; |0,0\rangle \rightarrow$ singlet $(s=0)$

$|1,-1\rangle, |1,0\rangle, |1,-1\rangle \rightarrow$ triplet $(s=1)$

For two identical spin-s particle:

no. of states $= (2s+1)^2$

$(s+1)(2s+1)$ symmetric

$s(2s+1)$ antisymmetric

$$\psi_{total} = \begin{cases} \psi^{symm.}_{total} = \begin{cases} \psi^{S}_{space} \times \psi^{S}_{spin} \\ \psi^{A}_{space} \times \psi^{A}_{spin} \end{cases} \\ \\ \psi^{anti}_{total} = \begin{cases} \psi^{S}_{space} \times \psi^{A}_{spin} \\ \psi^{A}_{space} \times \psi^{S}_{spin} \end{cases} \end{cases}$$

$$E_n^{(1)} = \frac{\langle n | \lambda (a^+ a^+)^2 | n \rangle}{\langle n | n \rangle}$$

$$= \lambda n^2 \frac{\langle n | n \rangle}{\langle n | n \rangle}$$

$$E_n^{(1)} = \lambda n^2 \qquad \boxed{C}$$

$\lambda (a^+ a a^+ a) |n\rangle$

$= \lambda \sqrt{n} (a^+ a a^+) |n-1\rangle$

$= \lambda (\sqrt{n})^2 (a^+ a) |n\rangle$

$= \lambda n \sqrt{n} (a^+) |n-1\rangle$

$= \lambda n^2 |n\rangle$

Que 6 Statement linked Questions:

An unperturbed two level system has energy eigen values E_1 and E_2 and eigen function $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ when perturbed its Hamiltonian is represented by:

$$\begin{pmatrix} E_1 & A \\ A^* & E_2 \end{pmatrix}$$

a) First order correction in E_1 is:

Sol^n:- $H_0 = \begin{bmatrix} E_1 & 0 \\ 0 & E_2 \end{bmatrix}$ & $H_p = \begin{bmatrix} 0 & A \\ A^* & 0 \end{bmatrix}$

$$|\phi_1\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad |\phi_2\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$E_1^{(1)} = \langle \phi_1 | H_p | \phi_1 \rangle = (1\ 0) \begin{bmatrix} 0 & A \\ A^* & 0 \end{bmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\boxed{E_1^{(1)} = 0}$$

b) second order correction to E_1 is:

Sol^n:- $$E_1^{(2)} = \sum_{m \neq n} \frac{|\langle \phi_m | H_p | \phi_n \rangle|^2}{E_n^{(0)} - E_m^{(0)}} = \frac{|\langle \phi_2 | H_p | \phi_1 \rangle|^2}{E_1 - E_2}$$

$$\langle \phi_2 | H_p | \phi_1 \rangle = (0\ 1) \begin{pmatrix} 0 & A \\ A^* & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = A^*$$

→ For Hard sphere scattering: $4\pi R^2$ $\pi R^2 = \sigma_T$

Q.M Classically

→ Rutherford scattering:

$$f(\theta) = \frac{-2m q_1 q_2}{4\pi\epsilon_0 \hbar^2 K^2} \simeq \frac{q_1 q_2}{16\pi\epsilon_0 E \sin^2(\theta/2)}$$

$$\frac{d\sigma}{d\Omega} = |f(\theta,\phi)|^2 = \left[\frac{q_1 q_2}{16\pi\epsilon_0 E \sin^2(\theta/2)}\right]^2$$

Diff. cross section

→ Partial wave analysis:

1) elastic case:

$$\sigma_{el} = \frac{4\pi}{K^2} \sum_{l=0}^{\infty} (2l+1) \sin^2\delta_l$$

$$f(\theta) = \frac{1}{K} \sum_{l=0}^{\infty} (2l+1)\, e^{i\delta_l} \sin\delta_l\, P_l(\cos\theta)$$

2) Inelastic Case:

$$\sigma_{in} = \frac{\pi}{K^2} \sum_{l=0}^{\infty} (2l+1)(1+n_l^2-2n_l\cos\theta)$$

$$\sigma_{total} = \sigma_{in} + \sigma_{el} \Rightarrow 2\pi a^2$$

High energy scattering of Black body

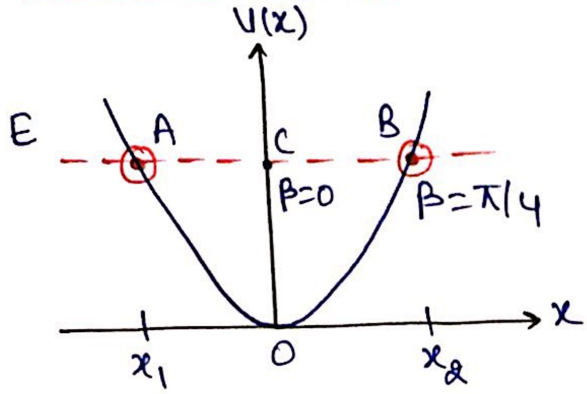
*** → valid only for symmetrical Potential (Central Potential)

WKB METHOD

→ Semi-classical Approximation method

1) Slowly varying Potential
2) in classical limit: $\lambda \to 0$ (condition)
3) used to calculate energy of system

Classical turning Points:



→ $\beta = \frac{\pi}{4}$: at A & B (Rigid)

→ $\beta = 0$: at C (Non-Rigid boundary)

at Points A and B:

$K.E = 0 \Rightarrow E_{total} = P.E$

$p = 0$

$$\lambda = \frac{h}{p} = \infty$$

→ WKB approximation method is valid in classical allowed and classical forbidden region.

→ WKB is not valid for turning point ($\lambda = \infty$, $p = 0$)

$$\left|\frac{d\lambda}{dx}\right| << 1$$

Shortcut method:

$V(x) \propto x^{2m}$

$$\boxed{E \propto n^{\frac{2m}{m+2}}}$$ ***